Tribhuvan University
Institute of Science and Technology

Course Title: Differential Equations
Course No. : Math 401
Level : B.Sc.
Nature of the Course: Theory

Full Marks: 100
Pass Mark: 35
Year: IV
Periods per week: 9

Course Objectives: The objective of this course is to acquaint students with the basic concepts of differential equation like first order linear and nonlinear differential equations, second order differential equations and higher order linear equations as well as partial differential equation with their wide range of applications in different fields. It aims at enabling students to build good knowledgebase in the subject of ordinary differential equations and partial differential equations.

Detailed Course

Units 1 , 2, 3, 4,5, 9 and 10 will be taught from Boyce and Diprima and units 6, 7 and 8 will be taught from Ian Sneddon

Unit 1: Introduction: 10 Lectures

1.1 Some mathematical models and direction fields: Modeling of falling objects, direction Field, Idea of constructing mathematical models
Problems: 1, 5, 7, 8, 9, 10, 11, 14, 15-20, 23, 24
1.2 Solutions of differential equations
Problems: 1a, 2a, 3, 8, 9, 12
1.3 Classification of differential equations,
Problems: 2, 4, 6, 7, 8, 12, 13, 14, 15, 16, 20, 21, 24, 25

Unit 2: First Order Linear and Nonlinear Differential Equations 15 Lectures

2.1 Integrating factors
Problems: 1c, 2c, 6c, 8c, 15, 17, 19, 20, 31, 32
2.2 separable equations
Problems: 1, 4, 5, 7, 8, 11, 16, 17, 19, 23, 26
2.3 Modeling with first order equations
Problems: 1, 2, 3, 4, 7, 12, 16, 19
2.4 Difference between Linear and Nonlinear differential equations
Theorem 2.4.1(without proof), Theorem 2.4.2 (without proof)
Problems: 2, 3, 4, 6, 7, 8, 10, 13, 16, 27, 30
2.5 Autonomous equations and Population Dynamics (Stability Theory)
Problems: 1,2,3,5,9,10,15,18,22
2.6 Exact equations and Integrating Factors
Theorems 2.6.1(Statement Only)
Problems: 1, 2, 4, 6, 10, 13, 16, 27, 30
2.7 Numerical Approximations: Eulers method
Problems: 1, 2, 4, 5, 11a, 21
2.9 First order difference equations
Problems: 1, 3, 4, 6, 8

Unit 3: Second Order Linear Equations: 15 Lectures

3.1 Homogeneous Equations with constant coefficients.
Problems: 3, 6, 7, 10, 11, 12, 15, 17, 20
3.2 Solutions of linear homogeneous equations; the Wronskian
Theorem 3.2.1(statement only), Theorem 3.2.2(statement only), Theorem 3.2.3(statement only), Theorem 3.2.4(statement only), Theorem 3.2.5(statement only), Theorem 3.2.6(statement and proof)
Problems : 2,3,4,5,7,8,13,14,16,22,38,39
3.3 Complex roots of the characteristic equations.
Problems : 1,5,8,11,14,17,19,21,35,36,37
3.4repeated roots, reduction of order
Problems: 3,4,5,9,12,13,16,23,25,41,42,43
3.5 Non-homogeneous Equations; Method of undetermined coefficients.
Theorem 3.5.1 (With proof)
Theorem 3.5.2 (With proof)
Problems :1-6,13,15,17,29
3.6 Variation of Parameters
Theorem 3.6.1(no proof)
Problems: 2, 5, 7, 9, 13
3.7. Mechanical and electric vibrations
Problems: 2, 3, 5, 6, 8, 11, 12, 17, 18
3.8. Forced vibrations
1, 3, 5, 7, 9, 11a, 12

Unit 4: Higher Order Linear Equations: 15 Lectures

4.1. General Theory of n^{th} order Linear Equations
Theorem 4.1.1(no proof), Theorem 4.1.2(no proof), Theorem 4.1.3(no proof)
Problems: 2, 4, 5, 7, 8, 11, 13, 15
4.2 Homogeneous equations with constant coefficients
Problems: 1, 4, 11, 13, 14, 16, 18, 32, 35
4.3 Method of undetermined coefficients
Problems: 2, 3, 7, 10
4.4 Method of Variation of Parameters
Problems: 2, 3, 4, 13

Unit 5: System of First Order Linear Equations: 15 Lectures

7.1 Introduction
Theorem 7.1.1(No Proof), Theorem 7.1.2(No Proof)
Problems:1, 3, 5 , 7, 10, 11
7.2. Review of Matrices
No question in exam
7.3 System of Linear algebraic equations: Linear independence, Eigenvalues, Eigenvectors
Problems: 1,4,5,8,10,13,14,18,23,32
7.4 Basic Theory of first order linear equations
Theorem 7.4.1(No Proof), Theorem 7.4.2(No Proof), Theorem 7.4.3(No Proof), Theorem 7.4.4(No Proof)
Problems: 3, 5 ,6, 7

Unit 6: Ordinary Differential Equations in More than Two Variables: 15 Lectures

1.1 Surface and curves in three dimensions
Problems: 1,2
1.2 Simultaneous Differential Equations of the first order and the first degree in three variables; 1.3 Methods of solution of dx/p=dy/q= dz/r
Problems :1,2,3
1.4 Orthogonal trajectories of a system of curves on a surface
Problems :1,2,3,5
1.5 Pfaffian Differential forms and Equations
Theorem 2 (No Proof), Theorem 3 (With Proof), Theorem 4 (No Proof), Theorem 5 (No Proof), Theorem 6 (With Proof)
Problems: 1,2,3,4

Unit 7: Partial Differential Equations of the First Order: 20 Lectures

2.1 Partial Differential equations
2.2 Origen of first order partial differential equations
1a,1b,2a,2b,2c,2e
2.3 Cauchy's problem for first order equations
Theorem 1(No Proof)
2.4 Linear equations of the first order
Theorem 2 (With Proof), Theorem 3 (No Proof)
Problems : 1,2,3,4,5
2.5 Integral surfaces passing through a given curve
Problems :2,3,4,5
2.6 Surfaces orthogonal to a given system of surfaces
Problems: 1,2
2.10 Charpit's Methods
Problems :1,2,3,6,7
2.11Special types of first order equations
Problems :1,2,3,4,6

Unit 8: Partial Differential Equations of the Second Order: 15 Lectures

3.1 The origin of second order equations
Problems: 1, 2, 3, 4
3.4 Linear PDE with constant coefficients
Theorem 1(With Proof), Theorem 2 (With Proof)
Problems: 2a, 2b, 2c, 3
3.5 Equations with variable coefficients
Problems : 2,4,5
3.11 Nonlinear equations of the second order (Monge's method)
Problems: 1, 3, 4, 5

Unit 9: Partial Differential Equations and Fourier Series : 15 Lectures
10.1 Two point boundary value Problems
Problems: 1, 2, 4, 5, 10, 11, 14, 15, 18
10.2 Fourier series
Problems: 1, 4, 6, 9, 14, 16, 17, 18
10.3 The Fourier Convergence Theorem
Theorem10.3.1(No Proof)

Problems : 1, 3, 6, 13, 17
10.4 Even and odd functions
Problems: 2, 4, 5, 8, 11, 15, 16, 17, 24, 31, 33

Unit 10: Separation of Variables: 15 Lectures

10.5 Separation of variables; Heat conduction in a Rod
Problems 1, 2, 4, 6, 7, 9, 11, 12
10.6 Other heat conduction Problems
Problems : 1, 2, 4, 5, 6, 8
10.7 The wave equation: Vibration of an Elastic string
Problems :2a, 3a, 5a ,12
10.8 Laplace's equations
Problems : 2, 6a ,6b, 10a

Note: We also suggest to look at all the solved examples of the related topics of the textbooks.

Text/ Reference Books:

1. Boyce, W. and DiPrima, R.; *Elementary Differential Equations and BoundaryValue Problems*, 9th Ed., Wiley India.
2. Ian Sneddon; *Elements of Partial Differential Equations*, McGraw Hill International Editions.
3. James C. Robinson; *An Introduction to Ordinary Differential Equations*, Cambridge University Press.

Guidelines to the question setters

There will be 10 questions of 10 marks each. All the questions are compulsory. There will be **three** OR choices in any question number from the same unit. It is also suggested to put at least one modelling problem (application) in one of the questions. The examination period of Math 401 will be 3 hours.

On the basis of the guidelines mentioned, we enclose one set of model question for Differential Equations (Math 401)

MODEL QUESTION
Tribhuvan University

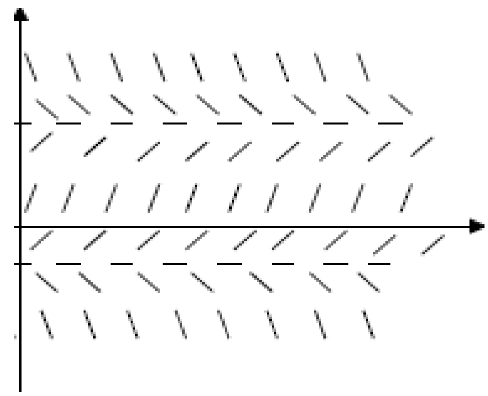
Bachelor Level / IV year/ Sc. & Tech. Full Marks: 100

Differential Equations (Math 401) Pass marks: 35

Time: 3 hrs

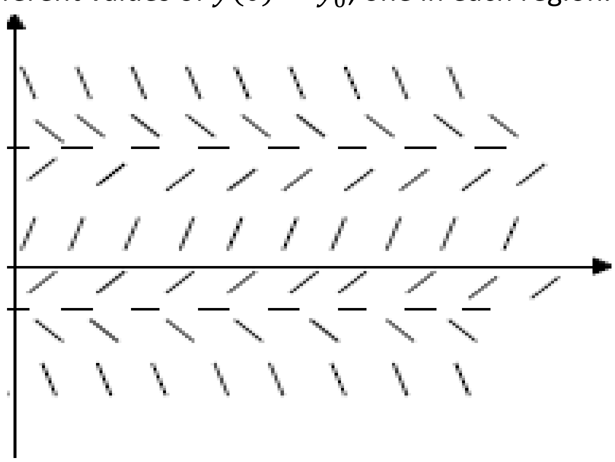
Attempt ALL the questions. Each question carries 10 marks.

1. Consider the slope field shown below.



   a. (3 points) Which of the following differential equations might have produced this slope field?
      i. $y' = (y-2)(y-6)$
      ii. $y' = (y+2)(y-6)$
      iii. $y' = (-y-2)(6-y)$
      iv. $y' = (y+2)(6-y)$

      Justify your answer.
   b. (3 points) Sketch at least 3 possible solutions curves for different values of $y(0) = y_0$, one in each region.



   c. (4 points) Determine the value of r for which the given differential equation has solutions of the form $y = t^r$ for $t > 0$.

$$t^2y'' + 4ty' + 2y = 0$$

2. Suppose a brine containing 0.2 kg of salt per liter runs into a tank initially field with 500L of water containing 5 kg of salt. The brine enters the tank at a rate of 5L/min. The mixture, kept uniform by stirring, is flowing out at the rate of 5L/min. Find the concentration of the salt in the tank after 10 minutes.
   a. (1 point) Write the appropriate variables with their units.
   b. (3 points) Construct a mathematical model of this flow process, that is, find the differential equation that describes this process
   c. (6 points) Find the concentration of the salt in the tank after 10 minutes.

OR

a. (3+1) Solve the initial value problem

$$y' = y^2, \quad y(0) = 1,$$

and determine the interval in which the solution exists.

b. (1+1+2+2) For the differential equation
$\frac{dy}{dt} = y(y-1)(y-2),$

sketch the graph of $f(y)$ versus y, determine the critical (equilibrium) points and classify each one as asymptotically stable, semistable or unstable Draw the phase line and sketch several graphs of solutions in the ty-plane.

3. a. (2+1+1) Verify that $y = 1$ and $y = t^{\frac{1}{2}}$ are solutions of the differential equation $yy'' + (y')^2 = 0$ for $t > 0$. Then show that $y = 1 + t^{\frac{1}{2}}$ is not a solution. Explain why this does not contradict the existence and uniqueness theorem or the principle of superposition.
b. (2+1) Find the Wronskian of the functions $y = cos^2 t$ and $y = 1 + \cos 2t$. Can these two functions form a fundamental set of solutions for second order differential equations?

c. (3 points) Without solving the problem, determine an interval in which the solution of the given initial value problem is certain to exist.

$$y' + (\tan t)y = \sin t, \qquad y(\pi) = 0$$

OR

(8+2) A spring is stretched 10 cm by a force of 2 Newtons. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 4 Newtons when the velocity is 1 m/sec. The motion of the mass is driven by an external force of 4 cos 2t Newtons. If the mass is initially at rest at equilibrium, find its position at any time t. Identify the transient and steady-state parts of the solution.

4. (10 points) Use the method of variation of parameters to determine the general solution of the given differential equation
$y''' - 2y'' - y' + 2y = e^{4t}$.
5. Consider the system
$x_1' = -2x_1 + x_2, \qquad x_2' = x_1 - 2x_2$

a. (2+4) Transform the system into a second order equation for x_1. Solve the equation for x_1 and then determine x_2 also.
b. (4 points) Find the solution of the given system that also satisfies the initial conditions $x_1(0) = 2$ and $x_2(0) = 3$.

6. (2+8) Define Pfaffian differential form and Pfaffian differential equation in n variables. Find the integral curves of the equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.
7. (3+7) Describe Charpit's method of solving the partial differential equation $f(x, y, z, p, q) = 0$ and use it to find a complete integral of the equation $p^2x + q^2y = z$.
8. (10 points) If $z = f(x^2 - y) + g(x^2 + y)$, where the functions f, g are arbitrary, prove that $\frac{\partial^2 z}{\partial x^2} - \frac{1}{x}\frac{\partial z}{\partial x} = 4x^2 \frac{\partial^2 z}{\partial y^2}$
9. (8+2) Assume that the function $f(x)$ defined by

   $f(x) = \begin{cases} -1, & -1 \le x < 0, \\ 1, & 0 \le x << 1 \end{cases}$

   is periodically extended outside the original interval. Find the Fourier series for the extended function. Also check whether the function $f(x) = \sec x$ is even, odd or neither.

   OR

   (6+4) Find the solution of the initial value problem with the periodic forcing term

   $$y'' + \omega^2 y = \sin nt, \quad y(0) = 0, \quad y'(0) = 0.$$

   Where n is a positive integer and $\omega^2 \neq n^2$. What happens if $\omega^2 = n^2$?
10. (10 points) Find the solution of the heat conduction problem

$$100u_{xx} = u_t, \qquad 0 < x < 1, \quad t > 0;$$
$$u(0, t) = 0, \qquad u(1, t) = 0, \quad t > 0;$$
$$u(x, 0) = \sin 2\pi x - \sin 5\pi x, \quad 0 \le x \le\le 1.$$